Analysis of Thermal Response in a Tissue Based on Thermal Wave Model

Chin-Tse Lin  
Dept. of Computer Application Engineering  
Far-East University  
Tainan, Taiwan 74448  
linchintse@feu.edu.tw

Kuo-Chi Liu  
Dept. of Mechanical Engineering  
Far-East University  
Tainan, Taiwan 74448  
kc@feu.edu.tw

Abstract—This paper would use the thermal wave model of bioheat transfer to analyze the thermal response in a perfused tissue subjected to a spatial heating that heat flux will decay exponentially with the distance from the heating surface. A modified discretization scheme based on the Laplace transform is proposed to solve the present problem. In analysis process, the surface convection effect is taken into account. The effects of perfusion rate and relaxation time on the behavior of bioheat transfer are discussed.

Keywords—bioheat transfer; perfused tissue; thermal wave model

I. INTRODUCTION

Present technology for superficial hyperthermia is based on external laser, microwave, or ultrasound techniques [1,2]. For treatment quality, spatial control of the temperature distribution is an absolute necessity. In accordance with the contents of the literatures [3-5], thermal behavior in nonhomogenous media needs a relaxation time to accumulate enough energy to transfer to the nearest element. The living tissues are highly nonhomogenous, and the velocity of heat transfer in tissues should be limited. To solve the paradox occurred in the classical Fourier’s law, Liu et al. [6] introduced the thermal wave model of bioheat transfer for investigation of physical mechanisms and the behaviors in thermal wave propagation in living tissues.

The rational prediction of temperature distribution is helpful to the development of hyperthermia. To take account the finite heat propagation effects for more realistic predictions, this work would employ the thermal wave model of bioheat transfer to analyzes the thermal behavior in perfused tissue induced by the heating source, which decays exponentially with the distance from the heating surface. As Lu et al. [7] described, the fundamental solution of the thermal wave model of bioheat transfer is extremely hard to obtain. Thus, a modified discretization scheme based on the Laplace transform method is developed. For prevention against burn injury in the health tissues near the skin surface, a surface cooling is often necessary during heating. Thus, the effect of surface convection on the thermal distribution is considered.

II. MATHEMATICAL FORMULATION

For more realistic predictions, Liu et al. [6] take account the finite heat propagation effects and derived the thermal wave model of bioheat transfer as

\[
\nabla \cdot (K \nabla T) + W_b C_b (T_b - T) + q_m + q_v + \tau \left( \frac{\partial T}{\partial t} + \frac{\partial q_v}{\partial t} - W_b C_b \frac{\partial T}{\partial t} \right) = \rho C \left( \tau \frac{\partial T}{\partial t} + \frac{\partial q_v}{\partial t} \right)
\]

where \( t \) is time, \( \rho \), \( C \), and \( T \) denote density, specific heat, and temperature of tissue. \( C_b \) and \( W_b \) are, respectively, the specific heat and perfusion rate of blood. \( q_m \) is the metabolic heat generation and \( q_v \) is the heat source for spatial heating. \( T_b \) is the arterial temperature. \( \tau \) is the relaxation time and is approximated as \( \tau = \alpha / V^2 \). \( \alpha \) is the thermal diffusivity and \( V \) denotes the heat propagation velocity in the medium.

In this paper, the one-dimensional bioheat transfer problem is analyzed, and (1) is simplified for constant thermal parameters and constant \( q_m \) with

\[
\frac{\partial^2 T}{\partial x^2} + W_b C_b (T_b - T) + q_m + \tau \left( \frac{\partial T}{\partial t} - W_b C_b \frac{\partial T}{\partial t} \right) = \rho C \left( \tau \frac{\partial T}{\partial t} + \frac{\partial q_v}{\partial t} \right)
\]

Under that laser, microwave, and ultrasound is employed to put heat on a tissue, the spatial heating \( q_v \) can be obtained as [8, 9]

\[
q_v (x, t) = \beta P(t) \exp(-\beta x)
\]

where \( P(t) \) is the time-dependent heating power on skin surface. \( \beta \) is the scattering coefficient.

To prevent the health tissues near the skin surface from burn injury, the forced surface cooling is always prosecuted in heating process. The boundary conditions for the present problem for \( t > 0 \) are described as

\[
q(0, t) = h_j (T_j - T)
\]

and

\[
\frac{\partial T(L, t)}{\partial x} = 0
\]

where \( h_j \) is the heat convection coefficient between the cooling medium and the skin surface, \( T_j \) the temperature of the cooling medium, and the location \( x = L \) is defined as the body core. The body core temperature \( T_c \) was often regarded as a constant.

The initial skin surface temperature and the core temperature were specified as 32.5 \(^\circ\mathrm{C}\) and 37 \(^\circ\mathrm{C}\), respectively. And then, the initial steady temperature distribution \( T_i (x, 0) \) in tissue can be computed with

\[
K \frac{\partial^2 T_i}{\partial x^2} + W_b C_b (T_b - T_i) + q_m = 0
\]

Subtracting (6) from (2) leads to