Bifurcation and Nonlinear Dynamic Analysis of Heart Blood Vessel System

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Abstract - Many studies have been shown and focused that variation of the quantity of vasoconstriction and blood pressure will cause the nonlinear chaotic behaviors in the heart blood vessel system and then induce the cardiovascular effect. Due to this kind of non-periodic motion is random and difficult to control, it is important to analyze and understand the status of dynamic system under different parametric conditions. In this paper, the differential transformation method is used to investigate the governing equations of system, and the dynamic behavior is characterized by reference to bifurcation diagrams, phase portraits, power spectra, and Poincaré map produced. The results indicate that the system behavior is significantly dependent on the magnitude of the vibrational amplitude. Specifically, the motion changes from T-periodic to 2T-periodic, then from 4T-periodic to 8-periodic, and finally to chaotic motion with windows of periodic motion as the vibrational amplitude is increased from 0.3 to 0.6. The results can be used as the basis for subsequent development of the control system design, and also reduced the possibility of cardiopathy.

Index Terms - Vasoconstriction, Blood pressure, Bifurcation diagram.

I. INTRODUCTION

Arrhythmia represents a potential of serious heart disease, stroke or sudden crisis of cardiac death. It is caused by the change of heartbeat rhythm, and then disturbed the contractive motions, meanwhile reduced the efficiency of transmission of blood and made human body great discomfort. There are many reasons for the rhythm change including the abnormalities of the heart ectopic pacemaker, and irregular dynamic phenomena such as the frequency of the conduction system. Pre-diagnosis of arrhythmia can allow patients to select the appropriate anti-arrhythmic drugs, thus improving the arrhythmia and other issues, while reducing the probability of sudden heart problems. Recently, medical instrument with experimental data is used to detect the electrocardiogram (ECG), and has become the major method for the diagnosis of disorders of cardiac rhythm. Also, it is applied to improve the accuracy of the detection of ECG and obtain the record of the heart activity [1].

The ECG detection is the most important part and also the first step of the cardiac examination. Each heartbeat is generated by the current and trigger from a specific group of cells in the heart blood vessel (HBV) system. ECG is detected the current to record signals, and thus used to evaluate for the status of the patient’s HBV system [2].

Coronary heart disease has become one of the global main cause of death, and been paid much attention by researchers, especially for the pathological changes of coronary artery disease and heart failure [3]. However, the electrocardiogram is a very important biological signal for the diagnosis of heart disease, but the signal is a kind of nonlinear data of the human body not easy to be analyzed. If the system can be monitored effectively and even controlled, perhaps heart disease can be inhibited. So, more and more scientists devoted the study of detection of abnormal ECG signals. In the past decade, several types of arrhythmia monitoring methods were developed including Bayesian methods, heuristic methods, expert systems, the Markov model and artificial neural networks etc. [4].

ECG is provided with three basic waveforms comprise the P wave, QRS wave and T wave, where P-wave generated in the far-field-induced specific electrical phenomena caused by the waveform of the heart surface [5]. These waves are the main factors to cause the movement of heart, so it is necessary to analyze the dynamic behaviors to be able to thoroughly understand the whole picture of HBV system [6]. If the cardiac motion can be precisely controlled, the heart disease and coronary artery or cardiac abnormal situation can be accurately diagnosed and cured [7].

This paper will apply the Differential transformation method theory to investigate the nonlinear dynamic behavior under certain parameters for the heart blood vessel system, and use the phase portrait, power spectra, bifurcation diagram and Poincaré map to further verify whether the system exist the chaos phenomenon. Also, it is hoped that this paper can be used as the basis of the subsequent development of the system control design, thereby inhibiting the emergence of non-linear behavior or aperiodic movement.

II. THEORETICAL ANALYSIS

A. Mathematical modeling

The governing equations of HBV system were derived in 2002 by Guan [8] and transferred to obtain in 2006 by Gong et al. [9] through the corresponding conversion as follows:

\[ \dot{x} + bx + cy = 0 \]  

(1)