A new algorithm for calculating two-dimensional differential transform of nonlinear functions

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\section*{1. Introduction}

The differential transform (one-dimensional) method first introduced by Zhou \cite{1} has been used in obtaining series solutions to a wide class of linear and nonlinear ordinary differential equations \cite{2,3,4,5,6,7,8,9,10}. Based on the same methodology, Chen and Ho \cite{12} recently developed the two-dimensional differential transform method for solving linear and nonlinear partial differential equations (PDEs). The main advantage of this method is that it can be applied directly to PDEs without requiring linearization, discretization or perturbation. Another important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate.

Although this newly emerged method has been proved to be an efficient tool for handling nonlinear PDEs \cite{12,13,14}, the nonlinear function \(f(u(x,y))\) used in these studies is restricted to the simple polynomial and derivative nonlinearities. For other more complicate nonlinearities such as exponential function, no related formula has been given to calculate their transform functions. This provides the motivation for the present work.

Recently, a new algorithm for calculating the one-dimensional differential transform of nonlinear functions has been established by the present authors \cite{14}. The developed technique depends only on the fundamental operation properties of differential transform and calculus. Based on the same concept, a simple and reliable algorithm used to calculate the two-dimensional differential transform of nonlinear functions in an easy way is introduced in this paper. The reliability and efficiency of the technique are demonstrated by solving three strongly nonlinear PDEs to obtain exact solutions for these problems.

\section*{2. Differential transform}

The basic definitions and fundamental \textbf{Theorems 1–5} of two-dimensional differential transform are defined and proved in \cite{11} and will be stated in brief in this section.