A Novel Method for mathematical modeling of Spatial Mechanisms with Spherical Joints

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Abstract—The kinematics of mechanical structures comprising links and joints are commonly analyzed using the Denavit-Hartenberg (D-H) notation. The current study presents an extended D-H notation which allows the independent parameters of any spatial binary mechanism, including one containing spherical pairs, to be derived for analysis and synthesis purposes. The validity of the proposed notation is demonstrated via its application to the analysis of typical RSCR mechanisms, where R, S and C denote revolute, spherical and cylindrical joints, respectively. The results confirm the viability of the extended D-H notation as a means of analyzing and enhancing the performance of arbitrary binary mechanisms comprising spherical pairs.

Keywords—spherical joint; D-H notation

I. INTRODUCTION

The D-H notation enables a simple mathematical representation of a link to be derived and is applicable to all mechanical systems comprising prismatic, revolute, helical and cylindrical joints. However, it can not be directly applied to the kinematic analysis of structures comprising spherical pairs since for this type of joint, the relative motion does not occur along one fixed axis. Accordingly, various researchers have proposed specific methods for the mobility analysis of spatial mechanisms with spherical joints. Yang [1] developed a joint-modeling matrix with a 3×3 dual-number form to analyze the displacement of spatial five-link mechanisms. Fischer [2] applied derivative-operator matrices to the displacement analysis of spatial mechanisms with ball joints. Gupta [3] developed a displacement equation to analyze the mobility region of PRSPR kinematic chains. The current study develops an extended D-H notation for the kinematic modeling of all spatial mechanisms, including those containing spherical joints.

II. MODELLING LINKS AND JOINTS

To analyze a mechanism containing a spherical joint, the matrix modeling the link terminating at the spherical joint can be embedded within a concatenated matrix comprising the transformation matrices of all the links and joints within the mechanism irrespective of their particular type.

A P-R link

In general, a P-R link may have prismatic, revolute, cylindrical and/or screw elements at its two ends. As shown in Figure 1, the link is characterized by two parameters, namely the link length $a_i$ and the twist angle $\alpha_i$, respectively, where $\alpha_i$ is the length of the common normal between the axes of the two joints and

$$\alpha_i = \theta_i - b_i$$

where $\theta_i$ is the angle between the two axes measured on a plane perpendicular to their common normal. The pose matrix of frame $(xyz)_i$ with respect to frame $(xyz)_{i-1}$ can be written as

$$A_i = \text{Rot}(z, \theta_i)\text{Trans}(0,0,b_i)\text{Trans}(a_i,0,0)\text{Rot}(x, \alpha_i)$$ (1)

Where $\text{Trans}(a_i,0,a_i)$ is the matrix corresponding to the translation vector $a_i+a_i+j+a_i$. Further, $\text{Rot}(x, \theta_i)_{i}$ are the rotation matrices about the x-axis. Note that in Eq. (1), $\theta_i$ is the angle between axes $x_{i-1}$ and $x_i$, measured on the plane normal to the axis of joint i, and $b_i$ is the distance between axes $x_{i-1}$ and $x_i$, measured along the axis of joint i. In this particular link, $\theta_i$ and $b_i$ are pair variables if joint i is a cylindrical joint.

B R-S link

Figure 2 presents an illustration of a typical R-S link comprising a revolute joint at one end and a spherical joint at the other. The link is characterized by the link length $a_i$, the axis of joint i and the center point $0_i$ of spherical joint $i+1$. The link length $a_i$ corresponds to the length of the common normal between the axes of the two joints. The frame associated with link i is mounted at $0_i$ and is positioned such that the $z_i$ axis is parallel to the axis of joint $i+1$ and passes through $0_i$, while the $x_i$ axis is aligned with the line from $0_{i-1}$ to $0_i$. Therefore, the pose matrix of frame $(xyz)_i$ with respect to frame $(xyz)_{i-1}$ can be written as

$$A_i = \text{Trans}(0,0,b_i)\text{Rot}(z, \theta_i)\text{Trans}(a_i,0,0)$$ (2)

As in the P-R link, $\theta_i$ is the angle between axes $x_{i-1}$ and $x_i$ and is measured on the plane normal to the axis of

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